8.2 Apply Exponent Properties Involving Quotients

WI **B.b.1** Compare, perform and explain operations on real numbers with and without context . . .

Before You used properties of exponents involving products.

Now You will use properties of exponents involving quotients.

Why? So you can compare magnitudes of earthquakes, as in Ex. 53.

Key Vocabulary
- **power,** *p. 3*
- **exponent,** *p. 3*
- **base,** *p. 3*

Notice what happens when you divide powers with the same base.

$$\frac{a^5}{a^3} = \frac{a \cdot a \cdot \cancel{a} \cdot \cancel{a} \cdot \cancel{a}}{\cancel{a} \cdot \cancel{a} \cdot \cancel{a}} = a \cdot a = a^2 = a^{5-3}$$

The example above suggests the following property of exponents, known as the quotient of powers property.

KEY CONCEPT *For Your Notebook*

Quotient of Powers Property

Let a be a nonzero real number, and let m and n be positive integers such that $m > n$.

Words To divide powers having the same base, subtract exponents.

Algebra $\frac{a^m}{a^n} = a^{m-n}, a \neq 0$ **Example** $\frac{4^7}{4^2} = 4^{7-2} = 4^5$

EXAMPLE 1 Use the quotient of powers property

SIMPLIFY EXPRESSIONS
When simplifying powers with numerical bases only, write your answers using exponents, as in parts (a), (b), and (c).

a. $\frac{8^{10}}{8^4} = 8^{10-4}$
$= 8^6$

b. $\frac{(-3)^9}{(-3)^3} = (-3)^{9-3}$
$= (-3)^6$

c. $\frac{5^4 \cdot 5^8}{5^7} = \frac{5^{12}}{5^7}$
$= 5^{12-7}$
$= 5^5$

d. $\frac{1}{x^4} \cdot x^6 = \frac{x^6}{x^4}$
$= x^{6-4}$
$= x^2$

GUIDED PRACTICE for Example 1

Simplify the expression.

1. $\frac{6^{11}}{6^5}$ **2.** $\frac{(-4)^9}{(-4)^2}$ **3.** $\frac{9^4 \cdot 9^3}{9^2}$ **4.** $\frac{1}{y^5} \cdot y^8$

POWER OF A QUOTIENT Notice what happens when you raise a quotient to a power.

$$\left(\frac{a}{b}\right)^4 = \frac{a}{b} \cdot \frac{a}{b} \cdot \frac{a}{b} \cdot \frac{a}{b} = \frac{a \cdot a \cdot a \cdot a}{b \cdot b \cdot b \cdot b} = \frac{a^4}{b^4}$$

The example above suggests the following property of exponents, known as the power of a quotient property.

KEY CONCEPT *For Your Notebook*

Power of a Quotient Property

Let a and b be real numbers with $b \neq 0$, and let m be a positive integer.

Words To find a power of a quotient, find the power of the numerator and the power of the denominator and divide.

Algebra $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$

Example $\left(\frac{3}{2}\right)^7 = \frac{3^7}{2^7}$

EXAMPLE 2 Use the power of a quotient property

SIMPLIFY EXPRESSIONS

When simplifying powers with numerical *and* variable bases, evaluate the numerical power, as in part (b).

a. $\left(\frac{x}{y}\right)^3 = \frac{x^3}{y^3}$

b. $\left(-\frac{7}{x}\right)^2 = \left(\frac{-7}{x}\right)^2 = \frac{(-7)^2}{x^2} = \frac{49}{x^2}$

EXAMPLE 3 Use properties of exponents

a. $\left(\frac{4x^2}{5y}\right)^3 = \frac{(4x^2)^3}{(5y)^3}$ Power of a quotient property

$= \frac{4^3 \cdot (x^2)^3}{5^3y^3}$ Power of a product property

$= \frac{64x^6}{125y^3}$ Power of a power property

b. $\left(\frac{a^2}{b}\right)^5 \cdot \frac{1}{2a^2} = \frac{(a^2)^5}{b^5} \cdot \frac{1}{2a^2}$ Power of a quotient property

$= \frac{a^{10}}{b^5} \cdot \frac{1}{2a^2}$ Power of a power property

$= \frac{a^{10}}{2a^2b^5}$ Multiply fractions.

$= \frac{a^8}{2b^5}$ Quotient of powers property

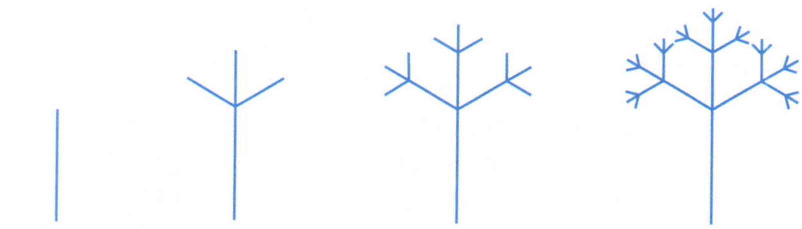
✓ GUIDED PRACTICE for Examples 2 and 3

Simplify the expression.

5. $\left(\frac{a}{b}\right)^2$ **6.** $\left(-\frac{5}{y}\right)^3$ **7.** $\left(\frac{x^2}{4y}\right)^2$ **8.** $\left(\frac{2s}{3t}\right)^3 \cdot \left(\frac{t^5}{16}\right)$

EXAMPLE 4 Solve a multi-step problem

FRACTAL TREE To construct what is known as a *fractal tree,* begin with a single segment (the trunk) that is 1 unit long, as in Step 0. Add three shorter segments that are $\frac{1}{2}$ unit long to form the first set of branches, as in Step 1. Then continue adding sets of successively shorter branches so that each new set of branches is half the length of the previous set, as in Steps 2 and 3.



a. Make a table showing the number of new branches at each step for Steps 1–4. Write the number of new branches as a power of 3.

b. How many times greater is the number of new branches added at Step 5 than the number of new branches added at Step 2?

Solution

a.

Step	Number of new branches
1	$3 = 3^1$
2	$9 = 3^2$
3	$27 = 3^3$
4	$81 = 3^4$

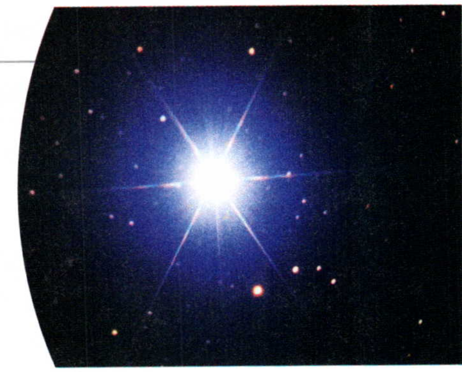
b. The number of new branches added at Step 5 is 3^5. The number of new branches added at Step 2 is 3^2. So, the number of new branches added at Step 5 is $\frac{3^5}{3^2} = 3^3 = 27$ times the number of new branches added at Step 2.

✓ GUIDED PRACTICE for Example 4

9. FRACTAL TREE In Example 4, add a column to the table for the length of the new branches at each step. Write the lengths of the new branches as powers of $\frac{1}{2}$. What is the length of a new branch added at Step 9?

EXAMPLE 5 Solve a real-world problem

ASTRONOMY The luminosity (in watts) of a star is the total amount of energy emitted from the star per unit of time. The order of magnitude of the luminosity of the sun is 10^{26} watts. The star Canopus is one of the brightest stars in the sky. The order of magnitude of the luminosity of Canopus is 10^{30} watts. How many times more luminous is Canopus than the sun?



Canopus

Solution

$$\frac{\text{Luminosity of Canopus (watts)}}{\text{Luminosity of the sun (watts)}} = \frac{10^{30}}{10^{26}} = 10^{30-26} = 10^4$$

▶ Canopus is about 10^4 times as luminous as the sun.

✓ GUIDED PRACTICE for Example 5

10. **WHAT IF?** Sirius is considered the brightest star in the sky. Sirius is less luminous than Canopus, but Sirius appears to be brighter because it is much closer to Earth. The order of magnitude of the luminosity of Sirius is 10^{28} watts. How many times more luminous is Canopus than Sirius?

8.2 EXERCISES

HOMEWORK KEY

○ = **WORKED-OUT SOLUTIONS** on p. WS18 for Exs. 33 and 51

★ = **STANDARDIZED TEST PRACTICE** Exs. 2, 19, 37, 46, and 54

◆ = **MULTIPLE REPRESENTATIONS** Ex. 49

SKILL PRACTICE

1. **VOCABULARY** Copy and complete: In the power 4^3, 4 is the __?__ and 3 is the __?__.

2. ★ **WRITING** *Explain* when and how to use the quotient of powers property.

EXAMPLES 1 and 2 on pp. 495–496 for Exs. 3–20

SIMPLIFYING EXPRESSIONS **Simplify the expression. Write your answer using exponents.**

3. $\frac{5^6}{5^2}$

4. $\frac{2^{11}}{2^6}$

5. $\frac{3^9}{3^5}$

6. $\frac{(-6)^8}{(-6)^5}$

7. $\frac{(-4)^7}{(-4)^4}$

8. $\frac{(-12)^9}{(-12)^3}$

9. $\frac{10^5 \cdot 10^5}{10^4}$

10. $\frac{6^7 \cdot 6^4}{6^6}$

11. $\left(\frac{1}{3}\right)^5$

12. $\left(\frac{3}{2}\right)^4$

13. $\left(-\frac{5}{4}\right)^4$

14. $\left(-\frac{2}{5}\right)^5$

15. $7^9 \cdot \frac{1}{7^2}$

16. $\frac{1}{9^5} \cdot 9^{11}$

17. $\left(\frac{1}{3}\right)^4 \cdot 3^{12}$

18. $4^9 \cdot \left(-\frac{1}{4}\right)^5$

19. ★ **MULTIPLE CHOICE** Which expression is equivalent to 16^6?

Ⓐ $\dfrac{16^4}{16^2}$ Ⓑ $\dfrac{16^{12}}{16^2}$ Ⓒ $\left(\dfrac{16^6}{16^3}\right)^2$ Ⓓ $\left(\dfrac{16^9}{16^6}\right)^3$

20. **ERROR ANALYSIS** *Describe* and correct the error in simplifying $\dfrac{9^5 \cdot 9^3}{9^4}$.

$$\frac{9^5 \cdot 9^3}{9^4} = \frac{9^8}{9^4} = 9^{12} \quad ✗$$

EXAMPLES 1, 2, and 3 on pp. 495–496 for Exs. 21–37

SIMPLIFYING EXPRESSIONS **Simplify the expression.**

21. $\dfrac{1}{y^8} \cdot y^{15}$ 22. $z^8 \cdot \dfrac{1}{z^7}$ 23. $\left(\dfrac{a}{y}\right)^9$ 24. $\left(\dfrac{j}{k}\right)^{11}$

25. $\left(\dfrac{p}{q}\right)^4$ 26. $\left(-\dfrac{1}{x}\right)^5$ 27. $\left(-\dfrac{4}{x}\right)^3$ 28. $\left(-\dfrac{a}{b}\right)^4$

29. $\left(\dfrac{4c}{d^2}\right)^3$ 30. $\left(\dfrac{a^7}{2b}\right)^5$ 31. $\left(\dfrac{x^2}{3y^3}\right)^2$ 32. $\left(\dfrac{3x^5}{7y^2}\right)^3$

33. $\left(\dfrac{3x^3}{2y}\right)^2 \cdot \dfrac{1}{x^2}$ 34. $\left(\dfrac{2x^3}{y}\right)^3 \cdot \dfrac{1}{6x^3}$ 35. $\dfrac{3}{8m^5} \cdot \left(\dfrac{m^4}{n^2}\right)^3$ 36. $\left(-\dfrac{5}{x}\right)^2 \cdot \left(\dfrac{2x^4}{y^3}\right)^2$

37. ★ **MULTIPLE CHOICE** Which expression is equivalent to $\left(\dfrac{7x^3}{2y^4}\right)^2$?

Ⓐ $\dfrac{7x^5}{2y^6}$ Ⓑ $\dfrac{7x^6}{2y^8}$ Ⓒ $\dfrac{49x^5}{4y^6}$ Ⓓ $\dfrac{49x^6}{4y^8}$

SIMPLIFYING EXPRESSIONS **Find the missing exponent.**

38. $\dfrac{(-8)^7}{(-8)^?} = (-8)^3$ 39. $\dfrac{7^? \cdot 7^2}{7^4} = 7^6$ 40. $\dfrac{1}{p^5} \cdot p^? = p^9$ 41. $\left(\dfrac{2c^3}{d^2}\right)^? = \dfrac{16c^{12}}{d^8}$

SIMPLIFYING EXPRESSIONS **Simplify the expression.**

42. $\left(\dfrac{2f^2g^3}{3fg}\right)^4$ 43. $\dfrac{2s^3t^3}{st^2} \cdot \dfrac{(3st)^3}{s^2t}$ 44. $\left(\dfrac{2m^5n}{4m^2}\right)^2 \cdot \left(\dfrac{mn^4}{5n}\right)^2$ 45. $\left(\dfrac{3x^3y}{x^2}\right)^3 \cdot \left(\dfrac{y^2x^4}{5y}\right)^2$

46. ★ **OPEN-ENDED** Write three expressions involving quotients that are equivalent to 14^7.

47. **REASONING** Name the definition or property that justifies each step to show that $\dfrac{a^m}{a^n} = \dfrac{1}{a^{n-m}}$ for $m < n$.

Let $m < n$. Given

$\dfrac{a^m}{a^n} = \dfrac{a^m}{a^n}\left(\dfrac{\frac{1}{a^m}}{\frac{1}{a^m}}\right)$ ___?___

$= \dfrac{1}{\frac{a^n}{a^m}}$ ___?___

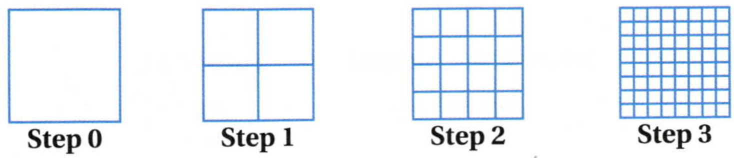
$= \dfrac{1}{a^{n-m}}$ ___?___

48. **CHALLENGE** Find the values of x and y if you know that $\dfrac{b^x}{b^y} = b^9$ and $\dfrac{b^x \cdot b^2}{b^{3y}} = b^{13}$. *Explain* how you found your answer.

PROBLEM SOLVING

EXAMPLES 4 and 5 on pp. 497–498 for Exs. 49–51

49. **MULTIPLE REPRESENTATIONS** Draw a square with side lengths that are 1 unit long. Divide it into four new squares with side lengths that are one half the side length of the original square, as shown in Step 1. Keep dividing the squares into new squares, as shown in Steps 2 and 3.



a. **Making a Table** Make a table showing the number of new squares and the side length of a new square at each step for Steps 1–4. Write the number of new squares as a power of 4. Write the side length of a new square as a power of $\frac{1}{2}$.

b. **Writing an Expression** Write and simplify an expression to find by how many times the number of new squares increased from Step 2 to Step 4.

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50. **GROSS DOMESTIC PRODUCT** In 2003 the gross domestic product (GDP) for the United States was about 11 trillion dollars, and the order of magnitude of the population of the U.S. was 10^8. Use order of magnitude to find the approximate per capita (per person) GDP.

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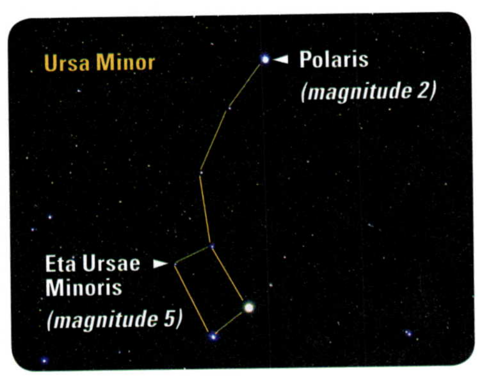
51. **SPACE TRAVEL** Alpha Centauri is the closest star system to Earth. Alpha Centauri is about 10^{13} kilometers away from Earth. A spacecraft leaves Earth and travels at an average speed of 10^4 meters per second. About how many years would it take the spacecraft to reach Alpha Centauri?

52. **ASTRONOMY** The brightness of one star relative to another star can be measured by comparing the magnitudes of the stars. For every increase in magnitude of 1, the relative brightness is diminished by a factor of 2.512. For instance, a star of magnitude 8 is 2.512 times less bright than a star of magnitude 7.

The constellation Ursa Minor (the Little Dipper) is shown. How many times less bright is Eta Ursae Minoris than Polaris?



53. **EARTHQUAKES** The energy released by one earthquake relative to another earthquake can be measured by comparing the magnitudes (as determined by the Richter scale) of the earthquakes. For every increase of 1 in magnitude, the energy released is multiplied by a factor of about 31. How many times greater is the energy released by an earthquake of magnitude 7 than the energy released by an earthquake of magnitude 4?

○ = WORKED-OUT SOLUTIONS on p. WS1 ★ = STANDARDIZED TEST PRACTICE ◆ = MULTIPLE REPRESENTATIONS

54. ★ **EXTENDED RESPONSE** A byte is a unit used to measure computer memory. Other units are based on the number of bytes they represent. The table shows the number of bytes in certain units. For example, from the table you can calculate that 1 terabyte is equivalent to 2^{10} gigabytes.

a. **Calculate** How many kilobytes are there in 1 terabyte?

b. **Calculate** How many megabytes are there in 1 petabyte?

c. **CHALLENGE** Another unit used to measure computer memory is a bit. There are 8 bits in a byte. *Explain* how you can convert the number of bytes per unit given in the table to the number of bits per unit.

Unit	Number of bytes
Kilobyte	2^{10}
Megabyte	2^{20}
Gigabyte	2^{30}
Terabyte	2^{40}
Petabyte	2^{50}

WI WISCONSIN MIXED REVIEW

TEST PRACTICE at classzone.com

55. For a car traveling at a speed of 45 miles per hour, the relationship between the distance traveled, d, and the time traveled, t, is described by the function $d = 45t$. Which statement is true?

Ⓐ The time traveled depends on the distance traveled.

Ⓑ The distance traveled depends on the time traveled.

Ⓒ The speed of the car depends on the distance traveled.

Ⓓ The speed of the car depends on the time traveled.

QUIZ *for Lessons 8.1–8.2*

Simplify the expression. Write your answer using exponents.

1. $3^2 \cdot 3^6$ *(p. 489)*
2. $(5^4)^3$ *(p. 489)*
3. $(32 \cdot 14)^7$ *(p. 489)*
4. $7^2 \cdot 7^6 \cdot 7$ *(p. 489)*
5. $(-4)(-4)^9$ *(p. 489)*
6. $\frac{7^{12}}{7^4}$ *(p. 495)*
7. $\frac{(-9)^9}{(-9)^7}$ *(p. 495)*
8. $\frac{3^7 \cdot 3^4}{3^6}$ *(p. 495)*
9. $\left(\frac{5}{4}\right)^4$ *(p. 495)*

Simplify the expression.

10. $x^2 \cdot x^5$ *(p. 489)*
11. $(3x^3)^2$ *(p. 489)*
12. $-(7x)^2$ *(p. 489)*
13. $(6x^5)^3 \cdot x$ *(p. 489)*
14. $(2x^5)^3(7x^7)^2$ *(p. 489)*
15. $\frac{1}{x^9} \cdot x^{21}$ *(p. 495)*
16. $\left(-\frac{4}{x}\right)^3$ *(p. 495)*
17. $\left(\frac{w}{v}\right)^6$ *(p. 495)*
18. $\left(\frac{x^3}{4}\right)^2$ *(p. 495)*

19. **AGRICULTURE** In 2004 the order of magnitude of the number of pounds of oranges produced in the United States was 10^{10}. The order of magnitude of the number of acres used for growing oranges was 10^6. About how many pounds of oranges per acre were produced in in the United States in 2004? *(p. 495)*

8.3 Zero and Negative Exponents

MATERIALS • paper and pencil

QUESTION How can you simplify expressions with zero or negative exponents?

EXPLORE Evaluate powers with zero and negative exponents

STEP 1 *Find a pattern*

Copy and complete the tables for the powers of 2 and 3.

Exponent, n	Value of 2^n
4	16
3	?
2	?
1	?

Exponent, n	Value of 3^n
4	81
3	?
2	?
1	?

As you read the tables from the *bottom up*, you see that each time the exponent is increased by 1, the value of the power is multiplied by the base. What can you say about the exponents and the values of the powers as you read the table from the *top down*?

STEP 2 *Extend the pattern*

Copy and complete the tables using the pattern you observed in Step 1.

Exponent, n	Power, 2^n
3	8
2	?
1	?
0	?
−1	?
−2	?

Exponent, n	Power, 3^n
3	27
2	?
1	?
0	?
−1	?
−2	?

DRAW CONCLUSIONS Use your observations to complete these exercises

1. Find 2^n and 3^n for $n = -3, -4,$ and -5.
2. What appears to be the value of a^0 for any nonzero number a?
3. Write each power in the tables above as a power with a positive exponent. For example, you can write 3^{-1} as $\frac{1}{3^1}$.